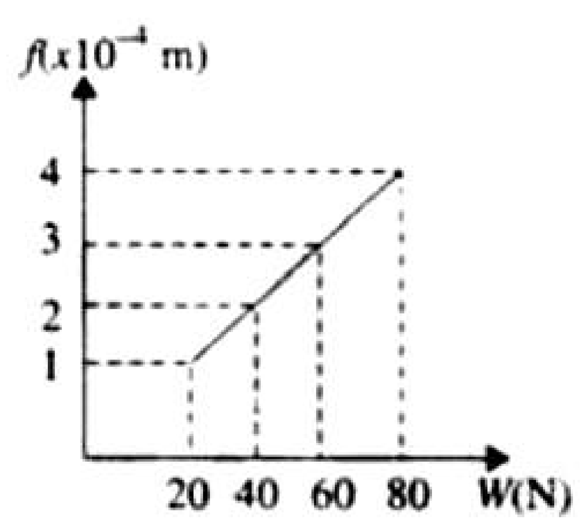
JEE Advanced

Single Correct Answer Type

1. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied?
   a. Length = 50 cm, diameter = 0.5 mm
   b. Length = 100 cm, diameter = 1 mm
   c. Length = 200 cm, diameter = 2 mm
   d. Length = 300 cm, diameter = 3 mm **(IIT-JEE 1981)**

2. Two rods of different materials having coefficients of thermal expansion and Young's moduli Y_1 and Y_2, respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\sigma_1 : \sigma_2 = 2 : 3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to
   a. 2 : 3 **b.** 1 : 1 **c.** 3 : 2 **d.** 4 : 9
   (IIT-JEE 1989)

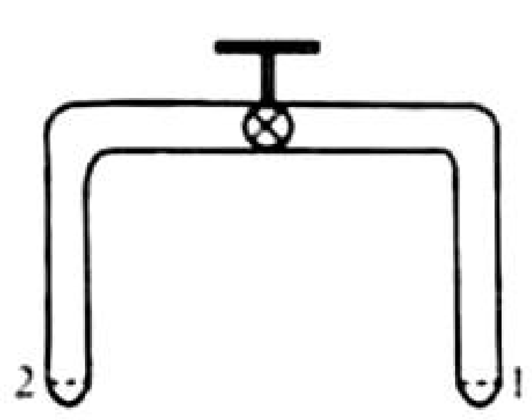
3. The adjacent graph shows the extension (Δf) of a wire of length 1 m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, Young's modulus of the material of the wire is



   a. 2×10^{11} N/m **b.** 2×10^{-11} N/m
   c. 3×10^{-12} N/m **d.** 2×10^{-13} N/m
   (IIT-JEE 2003)

4. When temperature of a gas is 20°C and pressure is changed from $p_1 = 1.01 \times 10^5$ Pa to $p_2 = 1.165 \times 10^5$ Pa, the volume changes by 10%. The bulk modulus is
   a. 1.55×10^5 Pa **b.** 0.115×10^5 Pa
   c. 1.4×10^5 Pa **d.** 1.01×10^5 Pa
   (IIT-JEE 2005)

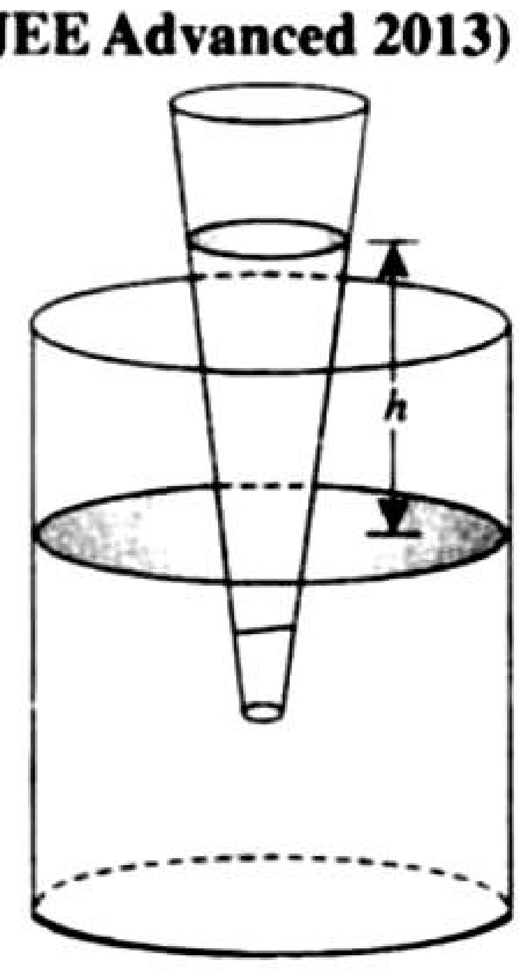
5. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in the figure. Just after opening the valve



   a. Air from end 1 flows towards end 2. There is no change in the volume of the soap bubble.
   b. Air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases.
   c. No change occurs.
   d. Air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases. **(IIT-JEE 2008)**

6. One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is
   a. 0.25 **b.** 0.50 **c.** 2.00 **d.** 4.00
   (JEE Advanced 2013)

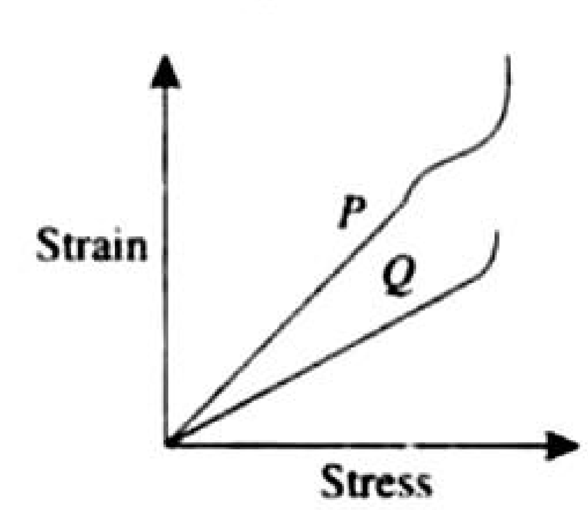
7. A glass capillary tube is of the shape of truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity)



   a. $\frac{2S}{b\rho g}\cos(\theta - \alpha)$ **b.** $\frac{2S}{b\rho g}\cos(\theta + \alpha)$
   c. $\frac{2S}{b\rho g}\cos(\theta - \alpha/2)$ **d.** $\frac{2S}{b\rho g}\cos(\theta + \alpha/2)$
   (JEE Advanced 2014)

Multiple Correct Answer Type

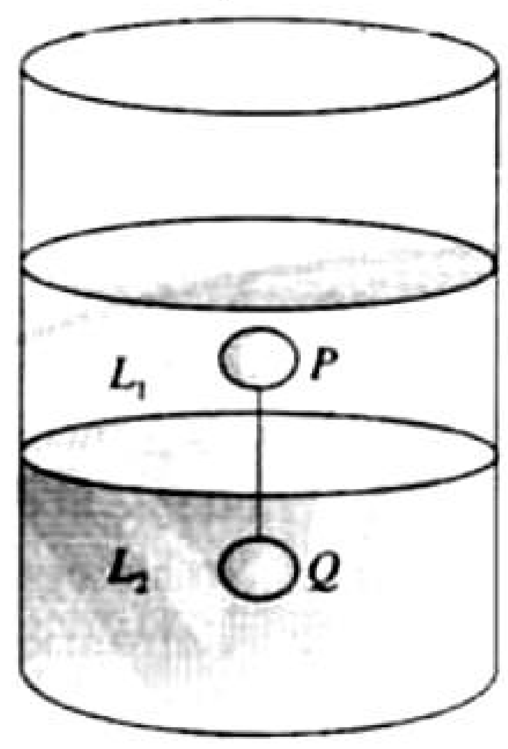
1. In plotting stress versus strain curves for the materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is(are)



a. P has more tensile strength than Q
b. P is more ductile than Q
c. P is more brittle than Q
d. The Young's modulus of P is more than that of Q.

(JEE Advanced 2015)

2. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then



a. $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$
b. $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_2}{\eta_1}$
c. $\vec{V}_P \cdot \vec{V}_Q > 0$
d. $\vec{V}_P \cdot \vec{V}_Q < 0$

(JEE Advanced 2015)

Linked Comprehension Type

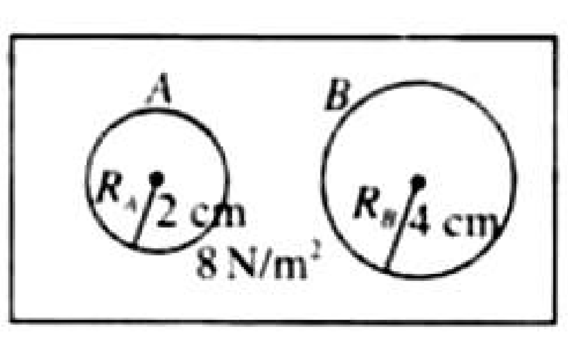
For Problems 1–3

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When the force becomes smaller than the weight of the drop, the drop gets detached from the dropper. **(IIT-JEE 2010)**

1. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r << R$) is
   a. $2\pi rT$
   b. $2\pi RT$
   c. $2\pi r^2T/R$
   d. $2\pi R^2T/r$
2. If $r = 5 \times 10^{-4}$m, $\rho = 10^3$ kg m^{-3}, $g = 10$ m s^{-12}, $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately
   a. 1.4×10^{-3} m
   b. 3.3×10^{-3} m
   c. 2.0×10^{-3} m
   d. 4.1×10^{-3} m
3. After the drop detaches, its surface energy is
   a. 1.4×10^{-6} J
   b. 2.7×10^{-6} J
   c. 5.4×10^{-6} J
   d. 8.1×10^{-6} J

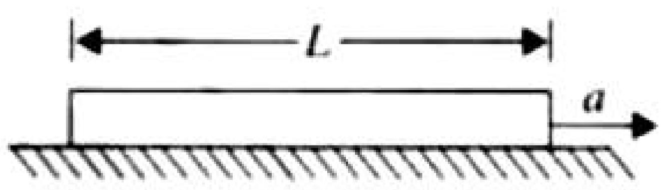
Integer Answer Type

1. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap water used to make bubbles is 0.04 N/m. Find the ratio nB/nA, where nA and nB are the number of moles of air in bubbles A and B, respectively. (Neglect the effect of gravity.) **(IIT-JEE 2009)**



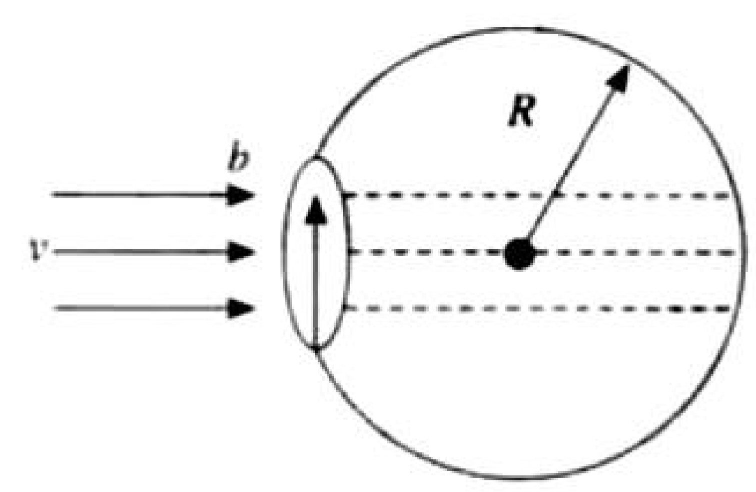
Fill in the Blanks Type

1. A wire of length L and cross-sectional area A is made of a material of Young's modulus. If the wire is stretched by an amount x, the work done is ____. **(IIT-JEE 1987)**
2. A solid sphere of radius R made of a material of bulk modulus K is surrounded by a liquid in cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass M is placed on the piston to compress the liquid, the fractional change in the radius of the sphere, $\delta R/R$, is ____. **(IIT-JEE 1988)**
3. A uniform rod of length L and density ρ is being pulled along a smooth floor with a horizontal acceleration a (see figure). The magnitude of the stress at the transverse cross section through the mid-point of the rod is____. **(IIT-JEE 1993)**



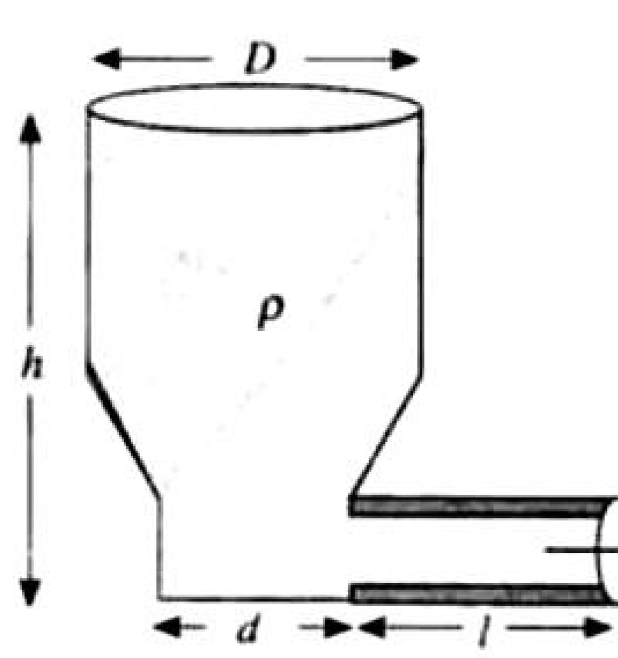
Subjective Type

1. A soap bubble is being blown at the end of very narrow tube of radius b. Air (density ρ) moves with a velocity v inside the tube and comes to rest inside the bubble. The surface tension of the soap solution is T. After sometime the bubble, having grown to radius r separates from the tube. Find the value of r. Assume that $r >> b$ so, that you can consider the air to be falling normally on the bubble's surface.



(IIT-JEE 2003)

2. A liquid of density 900 kg/m^3 is filled in a cylindrical tank of upper diameter 1.8 m and lower diameter 0.6 m. A capillary tube of length ℓ is attached at the bottom of the tank as shown in the figure. The capillary has outer radius 0.002 m and inner radius a. When pressure p is applied at the top of the tank volume flow rate of the liquid is 8×10^{-6} m^3/s and if capillary tube is detached, the liquid comes out from the

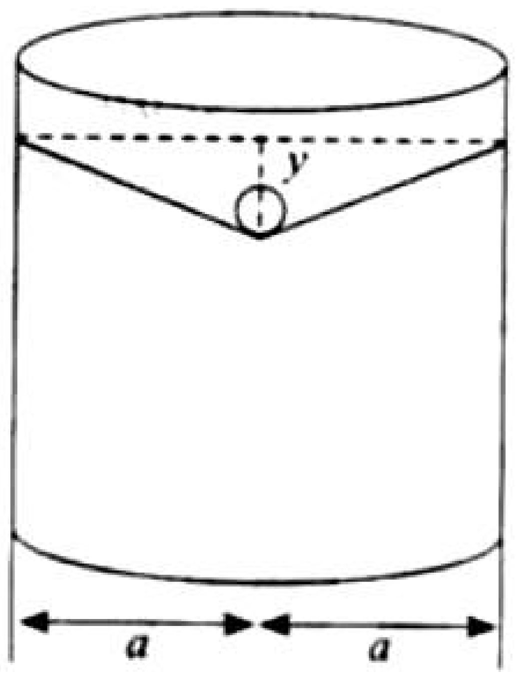


tank with a velocity 10 m/s. Determine the coefficient of viscosity of the liquid.

$\left[\text{Given: } \pi a^2 = 10^{-6}\ \text{m}^2 \text{ and } \frac{a^2}{l} = 2\times10^{-6}\ \text{m}\right]$

(IIT-JEE 2003)

3. A uniform wire having mass per unit length λ is placed over a liquid surface. The wire causes the liquid to depress by $y(y << a)$ as shown in the figure. Find surface tension of liquid. Neglect end effect. **(IIT-JEE 2004)**



4. In Searle's experiment, which is used to find Young's modulus of elasticity, the diameter of experimental wire is $D = 0.05$ cm (measured by a scale of least count 0.001 cm) and length $L = 110$ cm (measured by a scale of least count 0.1 cm). A weight of 50 N causes an extension of $X = 0.125$ cm (measured by a micrometer of least count 0.001 cm). Find maximum possible error in the values of Young's modulus. Screw gauge and meter scale are free from error. **(IIT-JEE 2004)**

5. A small sphere falls from rest in a viscous liquid. Due to friction, heat is produced. Find the relation between the rate of production of heat and the radius of the sphere at terminal velocity. **(IIT-JEE 2004)**

ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. a. **2.** c. **3.** a. **4.** a. **5.** b.
6. c. **7.** d.

Multiple Correct Answers Type

1. a., b. **2.** a., d.

Linked Comprehension Type

1. c. **2.** a. **3.** b.

Integer Answer Type

1. (6)

Fill in the Blanks Type

1. $\frac{YAx^2}{2L}$ **2.** $\frac{Mg}{3Ak}$ **3.** $\frac{1}{2}\rho a L$

Subjective Type

1. $\frac{4T}{\rho v^2}$

2. $\frac{1}{720}$ Ns/m^2

3. $\frac{\lambda a g}{2y}$

4. 1.09×10^{10} N/m^2

5. $\frac{dQ}{dt} \propto r^5$

HINTS AND SOLUTIONS

JEE Advanced

Single Correct Answer Type

1. a. $Y = \frac{T/A}{\Delta l/l}$

$$\Delta l = \frac{T \times l}{A \times Y} = \frac{T}{Y} \times \frac{l}{A}$$

Here, $\frac{T}{Y}$ is constant.

Therefore, $\Delta l \propto l/A$. As a result, l/A is largest in the first case.

2. c. $\frac{\text{Stress}}{\text{Strain}}$ and $\Delta l \propto l/A$

$\therefore$ Strain $\frac{\Delta l}{l}\ \alpha\Delta T \Rightarrow \text{Stress} = Y\alpha\Delta T$

For first rod, stress = $\gamma_1\alpha_1\Delta T$.
For the second rod, stress = $\gamma_2\alpha_2\Delta T$. Since stresses are equal.
Therefore,

$$\gamma_1\alpha_1\Delta T$$

$$\Rightarrow \quad \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$$

3. a. $Y = \frac{F}{A}\bigg/\frac{\Delta l}{l} = \frac{20 \times 1}{10^{-6} \times 10^{-4}} = 2 \times 10^{11}\ \text{N/m}^2$

4. a. $B = \frac{\Delta p}{\Delta v/V} = \frac{(1.165 \times 10^5 - 1.01 \times 10^5)}{0.1}$

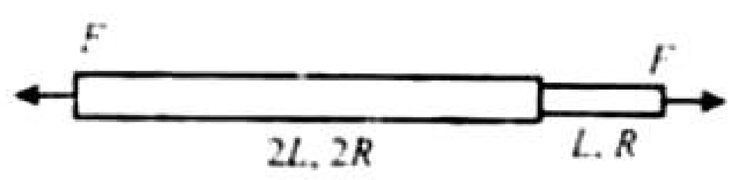
$$= 1.55 \times 10^5\ \text{Pa}$$

5. b. $\Delta p_1 = \frac{4T}{r_1}$ and $\Delta p_2 = \frac{4T}{r_2}$

$r_1 < r_2 \quad \therefore \Delta p_1 > \Delta p_2$

Therefore, air will flow from 1 to 2 and volume of bubble at end 1 will increase.

6. c.



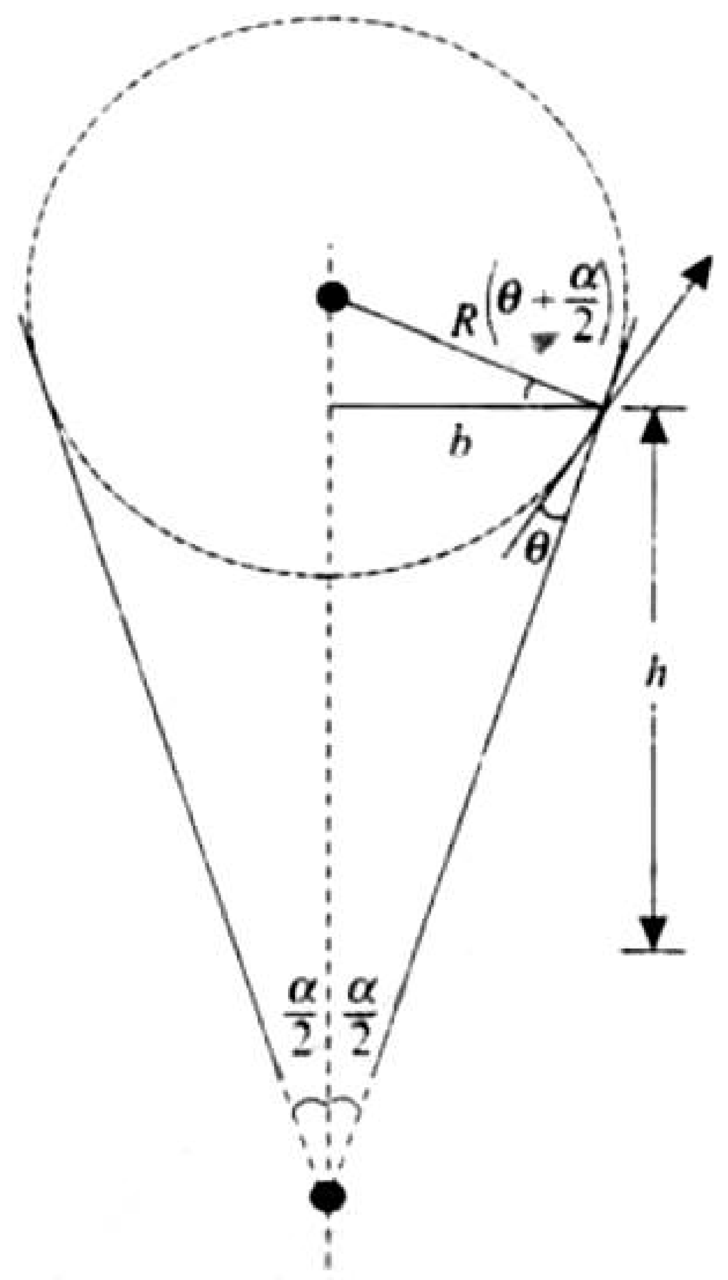
$$k_1 = \frac{\pi 4R^2 x}{2L},\ k_2 = \frac{\pi R^2 y}{L}$$

$$F = k_1 x = k_2 y$$

$$\Rightarrow \quad \frac{y}{x} = \frac{k_1}{k_2} = 2$$

7. d. If R be the meniscus radius

$$R\cos(\theta + \alpha/2) = b$$



Excess pressure on concave side of meniscus $= \frac{2S}{R}$

$$h\rho g = \frac{2S}{R} = \frac{2S}{b}\cos\left(\theta + \frac{\alpha}{2}\right) \Rightarrow h = \frac{2S}{b\rho g}\cos\left(\theta + \frac{\alpha}{2}\right)$$

Multiple Correct Answer Type

1. a, b.

As $(\text{Breaking stress})_P > (\text{Breaking Stress})_Q$

$\Rightarrow$ P is more tensile than Q.

We can increase the length of P by a larger amount than Q

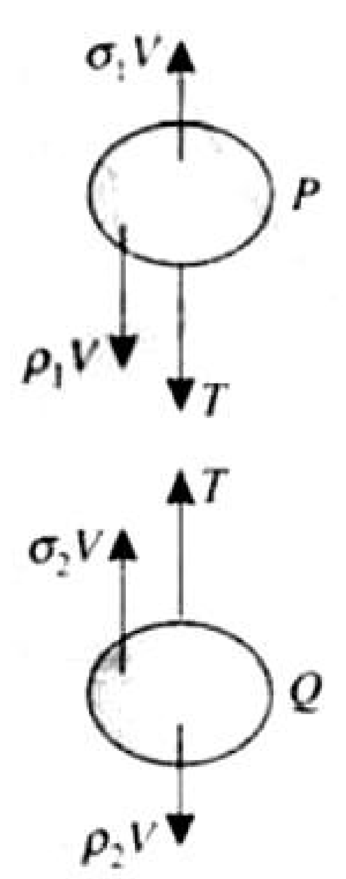
$\Rightarrow P$ is more ductile than Q.

$\Rightarrow P$ is less brittle than Q.

By checking the value of $\frac{\text{Stress}}{\text{Strain}}$ at any particular value of strain $\Rightarrow Y_Q > Y_P$

2. a., d.

Free body diagram of P and Q when they are connected by a string.



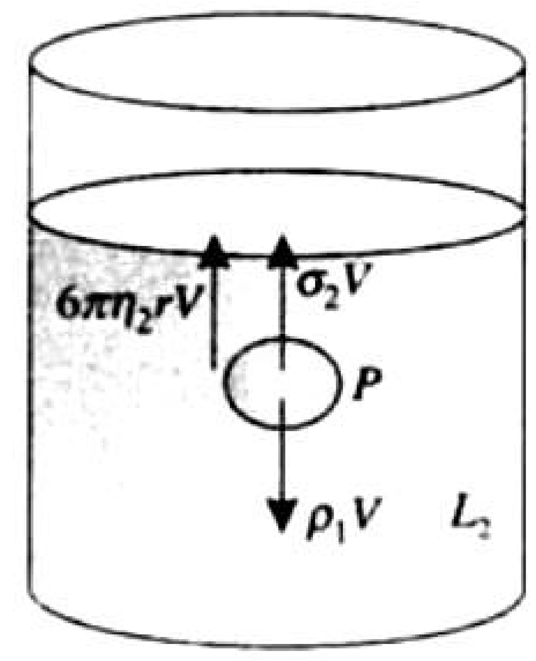
For P: $\sigma_1 V = \rho_1 V + T$ (i)

For Q: $T + \sigma_2 V = \rho_2 V$ (ii)

Adding (i) and (ii)

$$\sigma_1 - \rho_2 = \rho_1 - \sigma_2 \quad \text{(iii)}$$

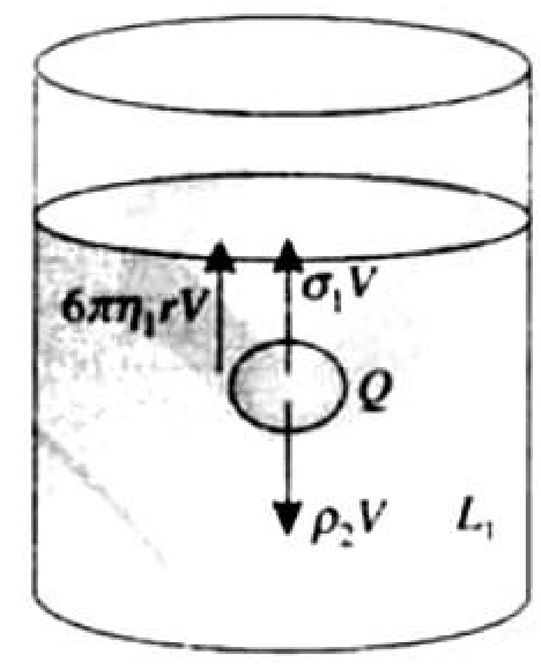
If the sphere P along in L_2



Terminal velocity of P: $V_P = \frac{2r^2}{9\eta_2}(\rho_1 - \sigma_2)$

If the sphere Q along in L_1

Terminal velocity of Q: $V_Q = \frac{2r^2}{9\eta_1}(\rho_2 - \sigma_1)$



$$\left|\frac{V_P}{V_Q}\right| = \frac{\eta_1}{\eta_2} \ \& \ \vec{V}_P \cdot \vec{V}_Q < 0$$

as if $\rho_1 - \sigma_2 > 0$

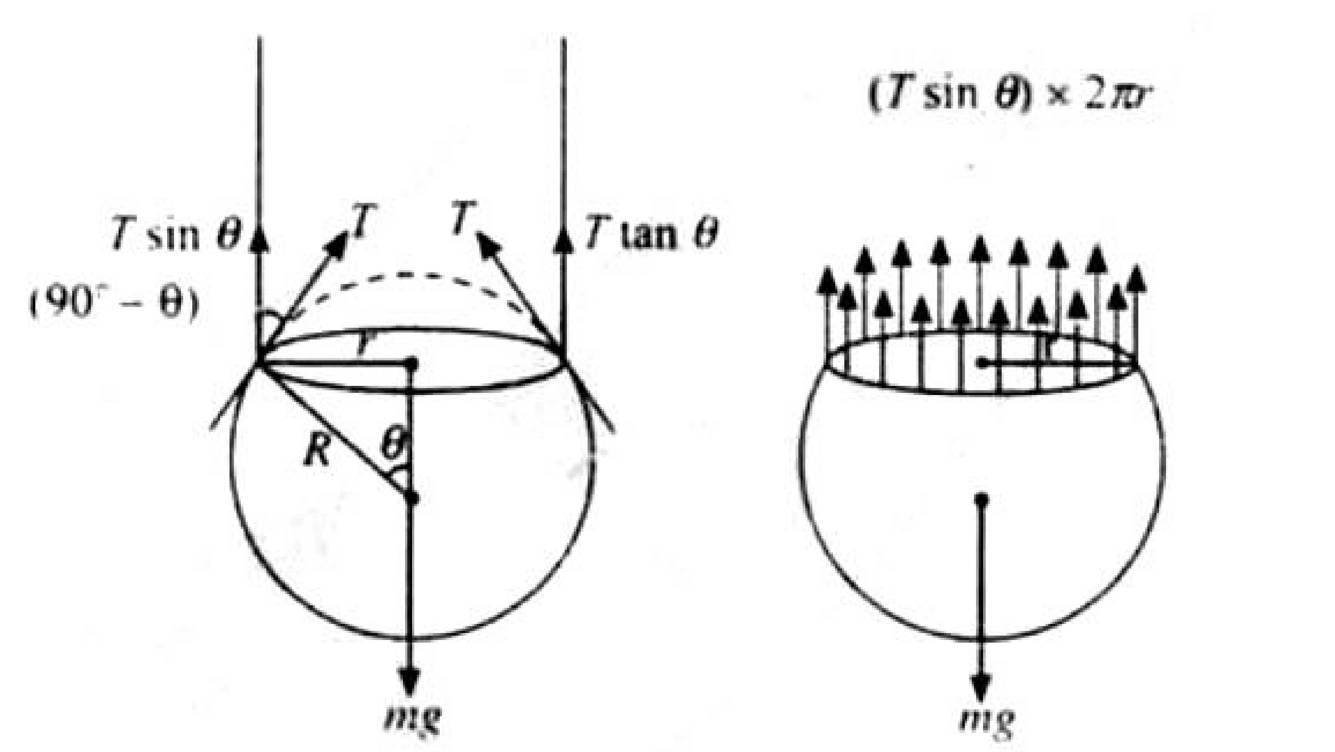
$\Rightarrow$ $\rho_2 - \sigma_1 < 0$

Integer Answer Type

1. c. From force body diagram of liquid drop, the vertical component of surface tension on the drop is

$$(F_{\text{surface}})_{\text{vertical}} = (T \sin\theta) - 2\pi r$$

$$= T \cdot 2\pi r \sin\theta = T \cdot 2\pi r \times \frac{r}{R} \Rightarrow (F_{\text{surface}})_{\text{vertical}} = \frac{2\pi r^2 T}{R}$$



2. a. The liquid drop detaches form the dropper when the vertical component of the force due to surface tension equals the weight of the drop. At this time

$$\frac{2\pi r^2 T}{R} = mg = \frac{4}{3}\pi R^3 \rho g$$

or $$R^4 = \frac{3}{2}\frac{r^2 T}{\rho g}$$

$$\Rightarrow \quad R = \left(\frac{3}{2}\frac{r^2 T}{\rho g}\right)^{1/4}$$

Substituting of the values we get

$R \simeq 1.4 \times 10^{-3}$ m

3. b. The surface energy of the drop

$E_{surface} = 4\pi R^2 \times T$

or $\quad E_{surface} = 4 \times \frac{22}{7} \times \pi \times (1.4 \times 10^{-3})^2 \times 0.11$

$\Rightarrow \quad E_{surface} = 2.7 \times 10^{-6}$ J

Integer Answer Type

1. (6) The excess of pressure above atmospheric pressure, due to surface tension in a bubble $\Delta p = \frac{4T}{r}$

The surrounding pressure for 1st bubble

$$P_A = P_0 + \frac{4T}{r_A} = 8 + \frac{4 \times 0.04}{0.02} = 16 \text{ N/m}^2$$

Similarly for 2nd bubble

$$P_B = P_0 + \frac{4T}{r_B} = 8 + \frac{4 \times 0.04}{0.04} = 12 \text{ N/m}^2$$

Using, $PV = nRT$

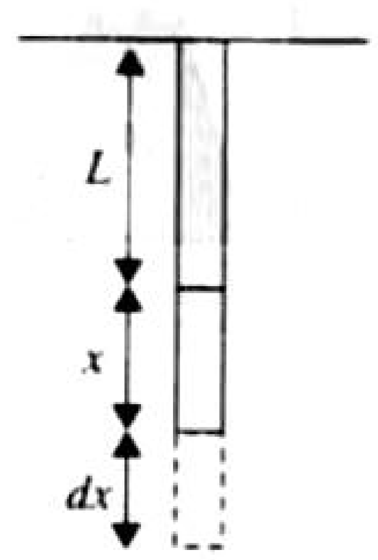
$$(16)\frac{4}{3}\pi(0.02)^3 = n_A RT \quad \text{(i)}$$

$$(12)\left(\frac{4}{3}\pi(0.04)^3\right) = n_B RT \quad \text{(ii)}$$

Dividing eq. (ii) with eq. (i) we get, $\frac{n_B}{n_A} = 6$

Fill in the Blanks Type

1. Let during the stretching of the wire, its extension at an instant of time is x and it is further stretched by dx. The restoring force acting at that instant is



$$Y = \frac{F/A}{x/L} \Rightarrow F = \frac{Yx}{1 \times L} \times A$$

The small amount of work done during the stretching by dx by the applied force

$$dW = Fdx = \frac{YxA}{L}dx$$

Total work done for stretching for distance x can be found by integrating above equation.

$$W = \int_0^x \frac{YxAdx}{L} = \frac{YA}{L}\int_0^x xdx = \frac{YAx^2}{2L}$$

2. $k = \frac{-\Delta p}{\Delta V/V}$; where $\Delta p = \frac{Mg}{A}$

$$\therefore \quad -\frac{\Delta V}{V} = \frac{Mg}{Al}$$

$$\Rightarrow \quad -\frac{(V_f - V_i)}{V_i} = \frac{Mg}{Ak}$$

$$\Rightarrow \quad -\frac{(V_i - V_f)}{V_i} = \frac{Mg}{Ak}$$

$$\Rightarrow \quad \frac{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi(R - \delta R)^3}{\frac{4}{3}\pi R^3} = \frac{Mg}{Ak}$$

$$\Rightarrow \quad \frac{R^3 - [R^3 - 3R^2\delta R]}{R^3} = \frac{Mg}{Ak}$$

$$\Rightarrow \quad \frac{\delta R}{R} = \frac{Mg}{3Ak}$$

3. Let A be the area of cross section of the rod.

$$\text{Mass } m \times \text{volume} \times \text{density} = \left(\frac{L}{2}A\right)\rho$$

$$T = ma = \left(\frac{L}{2}A\rho a\right)$$

Therefore, stress $\frac{T}{A} = \frac{1}{2}\rho a L$

Subjective Type

1. When the force due to excess pressure in the bubble equals the force of air striking at the bubble, the bubble will detach from the ring

$$\therefore \quad \rho A v^2 = \frac{4\pi}{r} \times A \Rightarrow r = \frac{4T}{\rho v^2}$$

2. When the tube is not there, using Bernoulli's theorem

$$P + P_0 + \frac{1}{2}\rho v_1^2 + \rho g H = \frac{1}{2}\rho v_0^2 + P_0$$

$$\Rightarrow \quad P + \rho g H = \frac{1}{2}\rho(v_0^2 - \rho v_1^2)$$

But according to equation of continuity $A_1 v_1 = A_2 v_2$ or $v_1 = \frac{A_2 v_0}{A_1}$

$$\therefore \quad P + \rho g H = \frac{1}{2}\rho\left[(v_0^2 - \left(\frac{A_2}{A_1}v_0\right)^2\right]$$

$$P + \rho g H = \frac{1}{2}\rho v_0^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

Here $P + \rho g H = \Delta P$

According to Poisseuille's equation $Q = \frac{\pi(\Delta P)a^4}{8\eta l}$

$$\therefore \quad \eta = \frac{\pi(\Delta P)a^4}{8Ql}$$

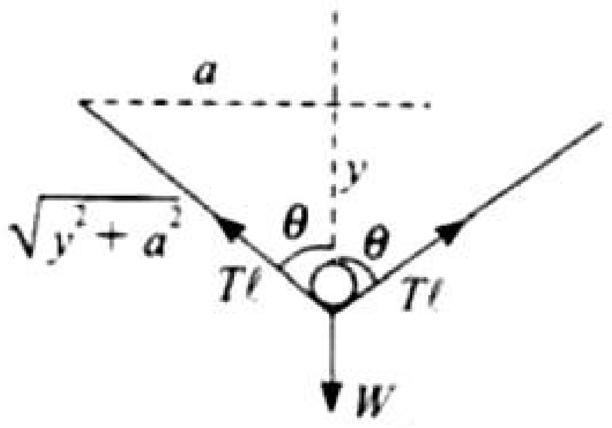
$$\therefore \quad \eta = \frac{\pi(P + \rho g H)a^4}{8Ql}$$

$$= \frac{\pi}{8Ql} \times \frac{1}{2}\rho v_0^2\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] \times a^4$$

Where $\quad \frac{A_2}{A_1} = \frac{d^2}{D^2}$

Substituting the value we get, $\eta = \frac{1}{720} \text{Ns/m}^2$

3. The free body diagram of wire is shown in the figure. Let ℓ be the length of wire. If λ is mass per unit length of wire, the weight of wire $W = (\ell x)g$ (acting vertically downward).
Vertical force due to surface tension $= 2T\ell \cos\theta$ (upward)
$\therefore$ For vertical equilibrium $2T\ell \cos\theta = (\ell\lambda)g$



$\Rightarrow \quad T = \frac{\lambda g}{2\cos\theta}$

From figure, $\cos\theta = \frac{y}{\sqrt{y^2+a^2}} \simeq \frac{y}{a}$ (for $y << a$ given)

$$T = \frac{\lambda g}{2\left(\frac{y}{a}\right)} = \frac{\lambda a g}{2y}$$

4. Maximum percentage error in Y is given by

$$Y = \frac{W}{\frac{\pi D^2}{4}} \times \frac{L}{X}$$

Substituting the values, we get

$$Y = \frac{50 \times 1.1 \times 4}{(1.25 \times 10^{-3}) \times \pi \times (5.0 \times 10^{-4})^2}$$
$$= 2.24 \times 10^{11} \text{ N/m}^2$$

$$\left(\frac{\Delta Y}{Y}\right)_{max} = 2\left(\frac{\Delta D}{D}\right) + \frac{\Delta x}{x} + \frac{\Delta L}{L}$$
$$= 2\left(\frac{0.001}{0.05}\right) + \left(\frac{0.001}{0.125}\right) + \left(\frac{0.1}{110}\right) = 0.0489$$

so maximum percentage error = 4.89%.

$$\Delta Y = (0.0489)Y = (0.0489) \times (2.24 \times 10^{11}) \text{N/m}^2$$
$$= 1.09 \times 10^{10} \text{ N/m}^2$$

5. Viscous force on a falling sphere in a liquid
$F = 6\pi\eta r v_t$

Where $v_t = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$ is terminal velocity.

ρ = density of sphere, σ = density of liquid.
Rate of production of heat (power) $= F.v_t$

$$P = 6\pi\eta r v_t^2 = 6\pi\eta r\left[\frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}\right]^2$$
$$= \frac{8}{27}\frac{\pi g^2(\rho-\sigma)^2}{\eta} r^5$$

Clearly $\quad \frac{dQ}{dt} \propto r^5$